CONSTRUCT ARGUMENTS: FRACTIONS AND GEOBOARDS

CONSTANCE RICHARDSON: Today, we want you to think about halves. And we're going to use the geoboards to show you – you're going to show me, actually – how many ways can we make half?

LINDA GOJAK: In this class, we see students actually trying to find half of a geoboard, and then they have to make conjectures about what would be the representation for banding off half of the geoboard and explaining how they know that it's half. The teacher really pushes their thinking and forces them to explain so they can't just say, "This is half." They have to really break it down into small pieces so they have convinced the other students in the class that what they are representing really is half of the area of the geoboard.

RICHARDSON: Now, if I wanted to take this geoboard and put it in half, what do you think I should do? The only way to get it in half was to cut it in half or put a rubber band around it.

RICHARDSON: Okay, all right, if I put it here, holding the rubber band down, I'm in half.

STUDENTS: No.

RICHARDSON: Why not? He said, "Put a rubber band around it and put it in half."

STUDENT: Put it more to the side a little bit.

RICHARDSON: You want it more to the side this way a little bit? Okay. Now we're at half.

STUDENTS: No.

You see what I mean? When I say "a half," I need to clearly understand what I mean by half. Are we at a half yet?

STUDENTS: No.

RICHARDSON: How do you know that, PJ?

STUDENT: Because there’s, um, five rows of pegs up there, and if you put it in the third, there’s going to be two on each side.

RICHARDSON: Oh, so we’re thinking a little bit. You counted there are five rows of pegs here, and you said if I put it on the third one?

STUDENT: The third one going down.

RICHARDSON: Third one going down, such as this... That’s half, and then you put another one around the third.

RICHARDSON: And put another one around the other third. And... are we at half?

STUDENTS: Yeah.

RICHARDSON: How could we prove that without just counting in the middle? They’re both the same size, or you could use a ruler.

RICHARDSON: All right, Corey says they’re both the same size. Can anyone see a shape in here that we could use to actually measure? A square.

RICHARDSON: A square, that’s right.

RICHARDSON: Is that a square?

STUDENTS: Yeah.

RICHARDSON: Could we use that shape to tell us if this is equal?

STUDENTS: Yeah.

RICHARDSON: All right. How many squares would fit in each of them, and see if one has more than the other. I could see how many squares would fit in each of them and see if one has more than the other. If this is truly a half, how many squares should appear in the other half?

STUDENTS: Eight.

RICHARDSON: Eight, you guys think so? As a group, you think so?

STUDENTS: Yeah.

RICHARDSON: Is there another way you can show me a half? You can go to the corner and go down diagonally.

STUDENT: You'll get halves of squares because it's in a triangle, and if you try making squares through it...

RICHARDSON: Well, let's just try it. Oh, no straight lines today. Half of squares, isn't that what you told me, PJ? Because, you said, the reason why...

STUDENT: Because if you've got a triangle... Right there, you just made one.

RICHARDSON: Ah, I just made one. Is that what you're talking about, PJ?

STUDENT: Yep.

RICHARDSON: Now, if we counted these – one, two, three, four, five, six-- we said half of a geoboard was equal to what?

STUDENTS: Eight squares.

RICHARDSON: Eight squares. If you put two of the halves together, then that would make a whole.

RICHARDSON: We could take this half and connect it to that one, is that what you're saying, PJ? And this half and connect it to that one, and we would get whole squares again, and we would be where?

STUDENTS: Eight.

RICHARDSON: Eight, half. All right.

GOJAK: I really like this task. I think it's a rich task, partially because there's a lot of different solutions. It's not just what you would think of by making a diagonal or cutting the geoboard in half. There are a lot of different ways to represent half on the geoboard, and so as kids come up with different ways or different solutions for the problem, they also have to justify to their classmates why that solution works.

RICHARDSON: You think this edge here is going to make one square?

STUDENT: Not as big as these, but you could make one.

RICHARDSON: Hmm, very interesting, huh? You think your theory might work? Are you beginning to wane on the theory, Billy? I think you're trying to give up on the pegs.

STUDENT: Can’t you do like PJ said and, like, put that with this and then make a square?

STUDENT 2: That’d make a triangle, though...

RICHARDSON: I don’t know. Talk to him.

STUDENT: It’d make a triangle, but these two together, you can, like, flip it around and make a square out of it.

RICHARDSON: I’m asking. Try it, take a look at it. See if you can record it on paper and count it. Dan’s solution sort of was like yours. He wanted to count pegs. Now, if you guys can prove to us that this is actually equal to eight squares... This one you say is only going to make one.

STUDENT: Mm-hmm, but it’s going to be bigger...

STUDENT 2: That’s not going to make a square. It’s going to make a triangle going like that.

STUDENT 3: If you flip these two around, it would be a square, but it would be bigger. It’d be the length of two of these if you flip these and put these together.

RICHARDSON: Marvin, do me a favor.

STUDENT: Yeah?

RICHARDSON: Use a rubber band and make this shape here.

STUDENT: See, that's what I was trying to say. It would be bigger than these squares right here.

RICHARDSON: See that shape? Can you try and make it for me? Is this shape the same as that one?

STUDENTS: Yeah.

RICHARDSON: Are these two shapes the same?

STUDENTS: Mm-hmm.

RICHARDSON: So Marvin, if you made the same shape on that side... Uh-oh, Billy's shaking his head now. His theory's working for him.

STUDENT: It's going to be bigger.

STUDENT: You'd have to put one coming down, put that there and you could put it around and still make a...

STUDENT: Marvin, no, just make it come across and down...

RICHARDSON: Let Marvin try and figure it out, okay? And then what?

STUDENT: Make it come down. Stretch it.

RICHARDSON: Ah... now.

STUDENT: It's as big as two of them.

RICHARDSON: Have we agreed that these two together make this?

STUDENTS: Yeah.

RICHARDSON: Now what does that prove?

STUDENT: That there's two.

RICHARDSON: That there's two. How many more did you need?

STUDENT: Two.

RICHARDSON: Is this half?

STUDENTS: Yes.

RICHARDSON: Good luck, guys.

When we think about the first three standards for mathematical practice, I don't think that there are three discrete standards. I don't think you can reason and make sense out of problems without thinking about them abstractly and quantitatively. And in order to build all that up, I think you really need to also give kids the opportunity to construct arguments – that is, to explain their thinking, justify their thinking to others, and have kids learn how to have mathematical conversations with each other, so that if I don't understand your thinking, you can explain to me and convince me why your reason or your answer is correct.